Use of Continuous Wavelet Transform-based tools for studying turbulence, aerodynamic forces and coherence

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Abstract. Fourier Transform and its tools have been applied popularly in many engineering applications. However, the Fourier Transform cannot analyze non-stationary, non-linear and intermittent signals with simultaneously time-frequency plane. Wavelet Transform have been invented recently to cope with limitations of the Fourier Transforms. This paper presents Continuous Wavelet Transform and its recent applications for analyzing experimental discrete data, especially for studying wind turbulence, aerodynamic forces and coherence in the time-frequency plane. Hidden high-energy event characteristics in the turbulence and aerodynamic forces as their correlation have been investigated in time-frequency plane using some advanced wavelet transformed-based tools not only wavelet coefficient as usual but only wavelet auto spectrum, wavelet coherence. Analyzing data of the turbulence and induced aerodynamic forces have been measured directly on physical models thanks to wind tunnel experiments.

Keywords: Continuous Wavelet Transform, wind tunnel test, turbulence, aerodynamic forces, coherence

1. Introduction

Wavelet Transforms (WT) is recently developed basing on a convolution operation between a signal and a basic wavelet function which allows to represent in time-scale (frequency) domains, also called as a time-frequency analysis [1]. WT advantages to conventional Fourier Transform (FT) and its modified version as Short-time Fourier Transform (STFT) in analyzing non-stationary, non-linear and intermittent signals with tempor-spectral information and multi-resolution concept. The WT uses the basic wavelet functions (wavelets or mother wavelets), which can dilate (or compress) and translate basing on two parameters: scale (frequency) and translation (time shift) to apply short windows at low scales (high frequencies) and long windows at high scales (low frequencies). Basing on a discretization manner of the time-scale plane and characteristics of wavelets, the WT can be mainly branched by the Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT), which they are completely different each other. Recently, the CWT has been applied to various topics in the wind engineering, and their applications still are evolving. The idea of using the CWT to study fluid turbulence was introduced by Farge 1992 [2], in which some new concepts such as local intermittency, wavelet power spectrum, decomposition energy were introduced with application potentials of the
wavelet transforms for turbulence analysis. Background and some applications of the CWT-based tools such as wavelet coefficient, wavelet spectra for meteorological data and geophysical turbulence were presented by Kumar and Fouloua-Georgiou 1997 [3]; Torrence and Compo 1998 [4]. Camussi and Guj 1997 [5] used orthogonal wavelets and the DWT to identify time information of coherent structures of wind turbulence. Bienkiewicz and Ham 1997 [6]; Jordan et al. 1997 [7] and Geurts et al. 1998 [8] detected cross correlation between turbulence and induced pressure in the time-frequency plane using the CWT-based wavelet spectral tool. Kareem and Kijewski 2002 [9] reviewed recent developments and future promise of the time-frequency analysis with emphasis in its applications to correlation detection and system identification, moreover, some higher-order CWT-based tools such as wavelet coherence and wavelet bi-coherence have been also introduced.

Random response of structures due to atmospheric turbulence has been based on typical models between turbulence (fluctuating velocity components) and turbulent-induced forces (buffeting forces). Because of the nature of random processes, the turbulence have been described in terms of statistical parameters such as turbulence intensity, turbulent scale, correlation functions, the FT-based spectral representation and tools. These quantities, however, are not time-dependent due to stationary assumption of turbulent signals. Up to now, existing relationship between turbulence and induced aerodynamic forces is based on the “quasi-steady theory” model [10]. This theory implies that the turbulent-induced aerodynamic forces are proportional to the instantaneously turbulence. Uncertainly, the quasi-steady theory is consistent if there is correspondence between the turbulent signals and turbulent-induced aerodynamic forces in the time domain and in the frequency domain. Some literatures [9][12] presented the natural existence of non-stationary characteristics of turbulence and the nonlinear correlation between turbulence and turbulent-induced pressure. Thus, further studies on cross correlation between turbulence and forces are needed, especially, to detect in the time-frequency plane in order to obtain better knowledge to refine existing problems from analytical computations to physical simulations.

This paper presents cored background and some recent applications of the CWT to investigate the wind turbulence, aerodynamic forces and to detect cross correlation of/between the turbulence and aerodynamic forces in the time-frequency plane. Some advanced tools such as the wavelet spectrum and wavelet coherence have been used. Analyzing experimental data of the turbulence and induced aerodynamic forces have been measured simultaneously and directly on rectangular cylinder in the wind tunnel.

2. Resolutions of time and frequency

Time signals contain explicitly only the time information, while the FT coefficient of the time signals exhibits the frequency information or explicitly in the frequency domain. Differences of time and frequency (Δt, Δf) stand for the resolution of time and frequency domains. Thus, the FT can exhibit perfectly for the frequency resolution of the time signals, but no time resolution (see Figure 1a). Resolutions of the time and frequency exhibit as the trade-off relationship, one can not obtain perfectly and simultaneously the time information and the frequency one. This can be understood as another expression of the Heisenberg Uncertainty Principle in the signal processing. In the FT, harmonic function associated with certain frequency prolongs over all length of the original signal, thus it means that no time information and time resolution can be tracked. In the STFT, known as modified FT or the Gabor transform, both time and frequency information can be obtained by using the window function and time translation. Length of the window function decreases the time resolution increases but the
frequency resolution reduces, or vice versa. This points out that the time resolution and the frequency resolution for all frequency and time instants become fixed for a certain fixed window. Thus, the STFT exhibits as equal resolution of time and frequency (see Figure 1b). In completely different way, the WT uses the wavelet function can either dilate or compress the window length and translate in the time shifts, which depend on frequency component. It can be explained that the wavelet function of long duration and low frequency is used for processing the low-frequency component of the original data, whereas, the one of short duration and high frequency for processing high-frequency component. Similar to the STFT, moreover, the wavelet function also translates along discrete time points to track the time information and the time resolution. Thus, concept of the multi-resolution of the time and the frequency as well as more effective technique of the discrete data processing can be obtained by the WT (see Figure 1c).

![Resolution map of time and frequency](image)

Fig 1. Resolution map of time and frequency

Thus, the CWT using the wavelet functions changed with different frequencies and different time durations, in which the wavelet function of low frequency and long duration used for low-frequency analysis, whereas the wavelet one of high frequency and short duration for high-frequency one.

3. Fourier transform and its tools

3.1. Definition

The continuous Fourier transform of continuous time signal \( X(t) \) is defined as the convolution operation between signal \( X(t) \) and harmonic function \( \phi(t) \) [11]:

\[
\hat{X}(f) = \int_{-\infty}^{\infty} X(t) \phi(t) dt = \int_{-\infty}^{\infty} X(t) \exp(-i2\pi ft) dt
\]

where \( \hat{X}(f) \) : Fourier transform coefficient (known as the first-order Fourier transform); \( f \) : Fourier frequency.

3.2. Fourier transform-based tools

The well-known Fourier transform-based second-order tools such as auto power spectral density (PSD) and cross power spectrum at/between two time signals are most used in the engineering applications due to their concern on the energy of the time signals, and defined respectively as:

\[
S_X(f) = E[\hat{X}(f)\hat{X}^\ast(f)]; \quad S_Y(f) = E[\hat{Y}(f)\hat{Y}^\ast(f)]; \quad S_{XY}(f) = E[\hat{X}(f)\hat{Y}(f)]
\]

(2)
where $S_X(f), S_Y(f), S_{XY}(f)$: auto power spectra of $X(t)$, $Y(t)$ and cross power spectrum between them; $E[]$: expectation operator; $^*,T$: complex conjugate and transpose operators; $\hat{X}(f), \hat{Y}(f)$: Fourier transform coefficients of time series $X(t), Y(t)$ respectively.

The FT-based coherence is approximately expressed as the normalized correlation coefficient of two spectral quantities of $X(t)$ and $Y(t)$ in the frequency domain:

$$COH_{XY}(f) = \frac{|S_{XY}(f)|}{\sqrt{S_X(f)S_Y(f)}}$$

(3)

where $|\cdot|$: absolute operator; $f$: Fourier frequency variable. The Fourier coherence is normalized between 0 and 1, thus two time signals $X(t), Y(t)$ are fully-correlated, coherence is unit, whereas coherence is zero, two time signals are uncorrelated in the frequency domain. Furthermore, linear phase coupling parameter between two time signals $X(t), Y(t)$ can be expressed as:

$$\Phi_{XY}(f) = \tan^{-1} \frac{\text{Im}[S_{XY}(f)]}{\text{Re}[S_{XY}(f)]}$$

(4)

where: Im, Re: Imaginary part and real one.

4. Continuous Wavelet Transform and its tools

4.1. Definition

The CWT of given signal $x(t)$ is defined as the convolution operation between signal $x(t)$ and wavelet function $\psi_{\tau,s}(t)$:

$$W^x(\tau,s) = \int_{-\infty}^{\infty} x(t) \psi^*_\tau,s(t)dt$$

(5)

where $W^x(s,\tau)$: CWT coefficients at translation $\tau$ and scale $s$ in the time-scale plane; asterisk $^*$ means complex conjugate; $\psi_{\tau,s}(t)$: wavelet function at translation $\tau$ and scale $s$ of basic wavelet function $\psi(t)$, or mother wavelet:

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi \left( \frac{t-\tau}{s} \right)$$

(6)

The mother wavelet, or wavelet for brevity, satisfy such following conditions as oscillatory function with fast decay toward zero, zero mean value, normalization and admissibility condition:

$$\int_{-\infty}^{\infty} \psi(t)dt = 0; \int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1; \quad C_\psi = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{\omega} d\omega < \infty$$

(7)

The CWT coefficients can be considered as a correlation coefficient and a measure of similitude between the wavelet and the signal in the time-scale plane. The higher coefficient is, the more the similarity. It is noted that the wavelet scale is not a Fourier frequency, but revealed as an inverse of frequency. Accordingly, a relationship between the Fourier frequency and wavelet scale can be approximated:

$$f_s = \frac{f_0 f_s}{s}$$

(8)

where $f_s$: Fourier frequency; $s$: wavelet scale; $f_0$: central frequency and $f_s$: sampling frequency.
4.2. Complex Morlet wavelet

Morlet wavelet and complex Morlet wavelet are commonly used for the CWT. The complex Morlet wavelet and its Fourier transform are given as follows:

\[
\psi(t) = (2\pi)^{-1/2} \exp(i2\pi f_0 t) \exp\left(-t^2 / 2\right) \tag{9a}
\]

\[
\tilde{\psi}(sf) = (2\pi)^{-1/2} \exp\left(2\pi^2 (sf - f_0)^2\right) \tag{9b}
\]

where \( f_0, s \): central frequency and wavelet scale of the complex Morlet wavelet.

Fig 2. Complex Morlet wavelet and its Fourier transform

The complex Morlet wavelet, however, is the most applicable for the CWT due to its containing of harmonic components and analogous to the Fourier transform (see Figure 2).

4.3. Measures of cross correlation detection

One would like to develop corresponding WT-based tools in the same manner with the FT’s ones that can detect the cross correlation of two signals in both frequency and time domains. Accordingly, wavelet cross spectrum at time shift index \( p \) and scale \( s \) of two signals \( x(t), y(t) \), having their respectively CWT coefficients \( W^x_p(s), W^y_p(s) \) is defined as follows:

\[
WCS^x_p(s) = \left\langle |W^x_p(s)W^y_p(s)| \right\rangle \tag{10}
\]

where \( \langle \rangle \) denotes smoothing operation in both time and scale directions; the wavelet auto spectra of \( x(t), y(t) \) are defined in the same manner as \( WPS^x_p(s) = \left\langle W^x_p(s)W^x_p(s)^* \right\rangle, WPS^y_p(s) = \left\langle W^y_p(s)W^y_p(s)^* \right\rangle \).

Similar to the Fourier coherence, the squared wavelet coherence of \( x(t), y(t) \) is defined as the absolute value squared of the smoothed wavelet cross spectrum, normalized by the smoothed wavelet auto spectra:

\[
WCO^x_p(s) = \frac{|\left\langle s^{-1}WCS^x_p(s) \right\rangle|^2}{\left\langle s^{-1} |WPS^x_p(s)| \right\rangle \left\langle s^{-1} |WPS^y_p(s)| \right\rangle} \tag{11}
\]

where \( |\rangle \) denotes the absolute operator; \( s^{-1} \) is used to convert to an energy density; by meaning \( 0 \leq WCO^x_p(s) \leq 1 \) in which wavelet coherence come to unit, two signals \( x(t), y(t) \) are full-correlated.
The wavelet phase difference between two signals in the time-frequency domain can be obtained as:

$$W\Phi^\delta_p(s) = \tan^{-1} \frac{\text{Im}\{s^{-i}WCS^\delta_p(s)\}}{\text{Re}\{s^{-i}WCS^\delta_p(s)\}}$$  \hspace{1cm} (12)

where Im, Re denote the smoothed imaginary and real parts.

### 4.4. Time and scale smoothing, end effect

Averaging in both time and scale directions must be required, especially in computing the wavelet spectrum and wavelet coherence. The averaging techniques of the wavelet power spectrum in time and scale at the time-shifted index \(p\) can be expressed as:

$$\left< WPS_p^2(s) \right> = \frac{1}{N_a} \sum_{p=p_1}^{p_2} |WPS_p(s)|^2 \quad \text{(Time smoothing)}$$  \hspace{1cm} (13a)

$$\left< WPS_p^2(s) \right> = \frac{\delta_j \delta_t}{C_\delta} \sum_{p=-l_1}^{l_2} |WPS_p(s_j)|^2 / s_j \quad \text{(Scale smoothing)}$$  \hspace{1cm} (13b)

where \(p\) assigned between \(p_1\) and \(p_2\); \(N_a\): number of averaged points (\(N_a = p_2 - p_1 + 1\)); \(\delta_j, \delta_t\): factor of window width and sampling period; \(C_\delta\): constant.

Because the CWT deals with finite-length signals, errors and bias values usually occur at two ends of signals, known as the end effect. One effective solution to eliminate the end effect is to truncate number of results at two ends of signals after the CWT is completed. Removed number, however, depend on the wavelet scale, thus so-called cone of influence should be estimated for more accuracy.

### 5. Wind tunnel test and experimental data

Physical measurements of the turbulence and the aerodynamic forces were carried out in the Kyoto University’s open-circuit wind tunnel. Rectangular section model with slenderness ratios B/D=5 was used in experiments. Model was rigidly fixed by support. Turbulent flows were artificially generated by the grid configuration in the wind tunnel at different mean wind velocities \(U=3\text{m/s}, 6\text{m/s}\) and \(9\text{m/s}\), corresponding to the flow case 1, case 2 and case 3. Intensities of turbulence are \(I_c=11.46\%\), \(I_c=11.23\%\) (case 1); \(I_c=10.54\%, I_c=9.28\%\) (case 2) and \(I_c=9.52\%, I_c=6.65\%\) (case 3). Two turbulence components: alongwind \(u(t)\) and acrosswind \(w(t)\) were measured by the X-probe hot-wire thermal constant anemometer (CTA): X-probe (Model 1011, Kanomax Inc., Japan), CTA and linearization (Model DC Voltmeter 1008, Nihon Kagaku Kogyo, Japan). Three aerodynamic force components: drag, lift and moment were measured by the dynamic multi-component loadcells (Model LMC 3505-30N, Nissho Electric Works, Co., Japan). Electric signals of measured forces are amplified by the 8-channel conditioner (Model DCM 8A, Kyowa Corp., Japan), then are filtered by 100Hz low-pass filters (E3201, NF Design Block Co., Ltd.). Signals of the turbulence and the forces are digitally sampled by A/D converter at the sampling rate 1000Hz over 100 seconds (Thinknet DF3422, Pavec Co., Ltd., USA). Experimental set-ups and instrumentation are shown in Figure 3.

Measured turbulence and induced aerodynamic forces have been used for the above-mentioned WT-based tools. Time series of the turbulence and aerodynamic forces at the flow case 1 are shown in Figure 4 and their Fourier power spectral densities at three flow cases in Figure 5.
Fig 3. Experimental set-ups and instrumentation

Fig 4. Time series of turbulence and aerodynamic forces at flow case 1

Fig 5. Fourier power spectral densities of turbulence and aerodynamic forces at three flow cases
6. Results and discussion

The CWT-based coherence has been used to detect the correlation structure between the turbulence and induced forces. The CWT coefficients, wavelet auto spectra and wavelet cross spectra of turbulence (u-, w-components) and induced lift, moment have been estimated in the first step before the wavelet coherence and the wavelet phase difference can be obtained with use of the complex Morlet wavelet.

Figure 6 shows the wavelet auto spectral maps of u-, w-turbulences and lift, moment on the time-frequency plane of 0–50Hz frequency band and 0–40 second interval in which 5-second intervals at two ends are cut off for eliminating the end-effect. These wavelet maps are presented with corresponding FT-based power spectral densities (PSD) in the frequency domain.

![Wavelet auto spectral maps and corresponding Fourier auto spectra of u-, w-turbulences and lift, moment](image-url)
It can be seen from Figure 6, some comments can be given. Firstly, the wavelet auto spectral maps distribute discretely and intermittently in both the time and frequency domain. Thus, intermittent power spectra are considered as the existing nature of the turbulences and induced forces. Secondly, the FT-based spectral tool seems to be perfect to detect exact frequencies of dominant spectral events, while the wavelet spectral tool is appropriate to track frequency band of these spectral events. However, there is no time information of spectral components obtained at any observed frequency in the Fourier spectra, but eventual time of spectral components can be given in the wavelet spectra. Thirdly, there is correspondence in the dominant spectral components between the wavelet spectra and the Fourier spectra, especially in cases when dominant spectral peaks exist in their signals. For example, dominant spectral peak at certain frequency $31.7\text{Hz}$ is clearly observed in the Fourier spectrum of the moment, and this peak occurs at frequency band $30\div35\text{Hz}$ in the wavelet spectral map.

![Wavelet coherence maps and corresponding Fourier coherences between u-, w-turbulences and lift, moment](image)
Event of this high power spectrum at this frequency do not occur continuously in whole time domain, but occurs intermittently at such localized time intervals as roughly 3±5s, 11±13s, 14s, 17s, 22s, 27±31s and 33s. Moreover, where the power spectra distribute broadly without dominant spectral peaks on the frequency domain, high spectral events locally distributed only can be detected by the wavelet spectral maps in the time-frequency plane. As cases of u-, w-turbulence, some local events can be observed around 28s, 31s and 11Hz in the wavelet spectral map of u-turbulence and around 16s and 10Hz in that of w-turbulence, these events cannot be detected by the Fourier spectra. Thus, it is advisable to use advantages of both Fourier spectra and wavelet one to track the high power spectral events of signals in the time-frequency plane. Finally, dominant spectral events of u-, w-turbulences and induced lift, moment do not occur simultaneously in the time-frequency plane.

Figure 7 indicates the temporo-spectral information of cross correlation between u-, w-turbulences and their induced lift, moment due to the wavelet coherence maps and corresponding Fourier coherences. Some following comments can be given from the Figure 7. Firstly, the cross correlation between the turbulences and aerodynamic forces also distributes intermittently and discretely like their wavelet auto spectra in the time-frequency plane. Thus, it is also discussed that the intermittent distribution of the cross correlation between the turbulences and their induced forces is observed as the nature of intermittency in the time-frequency plane. Secondly, while the Fourier coherences between the turbulences and aerodynamic forces exhibit small (ranging between 0 and 0.4) in the frequency domain, but high values of the cross correlation (even to be nearly unit at some local zones) still are observed and distributed locally in the time-frequency plane. This finding implies that the high coherence events between the turbulences and aerodynamic forces that are distributed intermittently in the time domain can not be observed by the Fourier coherence due to its averaging operation in the time domain, but are detected by the wavelet one. Finally, correspondence of high coherence events between the Fourier coherence and the wavelet one can be observed in some cases. For example, some high Fourier coherences are observed around 1.8Hz, 3Hz, 11Hz in the cross correlation of w-turbulence and lift, that are corresponding to high wavelet coherences occurring around 14s, 23s, 26s (at 1.8Hz), 23s, 26s (at 3Hz) and 21s, 26s (at 11Hz). Some high coherence events, however, are detected at local zones in the wavelet coherence maps, but do not appear in the Fourier coherence due to its time-domain averaging. For example, high wavelet coherence is detected locally around 10Hz and 7s in the correlation of u-turbulence and lift, but the Fourier coherence in this frequency exhibits very small.

7. Conclusion

The CWT and its tools have been applied for analyzing the time series signals, studying the wind turbulence and turbulence-induced aerodynamic forces, detecting coherence between them. Experimental data of the wind turbulence and aerodynamic forces have been extracted directly through the wind tunnel measurements. CWT-based results have been compared with corresponding FT-based ones in somewhere as needed. Some conclusions can be given as follows:

The CWT-based tools can detect the high energy events in the time series in the time-frequency plane. The CWT can detect what time periods that high spectral events occurred, which this cannot do with the conventional WT-based tools.

High energy events of the wind turbulence and induced forces do not occur simultaneously on the time domain at the same observed frequency bands. Prominent frequency bands of the wind
turbulences, moreover, that contribute dominantly on total energy seem to be different from those of the induced aerodynamic forces.

High coherence events between the wind turbulence and the aerodynamic forces are always observed in the CWT analysis, but localized in large or small time-frequency zones depending on certain cases. These findings may imply that the wind turbulence and aerodynamic force signals exhibit non-stationary and non-linear features.

Relationship between the wind turbulence and induced aerodynamic forces based on the quasi-steady theory is not cohesive, and conventionally spectral-based analytical tools might produce risks and uncertainties in the response prediction of structures. More consistent theory for the random response prediction of structures should be developed accounting for non-stationary, non-linear features and less cross correlation of the wind turbulence and the aerodynamic forces.

Acknowledgement. Physical data measurements of the wind turbulence and aerodynamic forces have been carried out at the wind tunnel of the Structural and Wind Engineering Laboratory, the Kyoto University, Japan. The author would acknowledge and express the special thank for this support.

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